NSFCAM Image Scale and Distortion Characteristics

based on material extracted from

Coordinated 1996 HST and IRTF Imaging of Neptune and Triton. I. Observations, Navigation, and Deconvolution Techniques. by L.A. Sromovsky, P.M. Fry, K.H. Baines, S.S. Limaye, G.S. Orton, and T.E. Dowling, submitted to *Icarus* on 14 July 1999.

Abstract

To obtain sub-pixel accuracy in navigation of IRTF images of Neptune, we needed to determine the NSFCAM pixel scale for each filter used and as a function of location on the detector array to better than 0.1%. This was accomplished using measurements of Uranian satellites and stellar angular separations at different locations on the detector array. When set to the nominal value of 0.148″/pixel we found that the NSFCAM image scale varied with filter from 0.1470″/pixel to 0.1479″/pixel at the center of the image and increased near the edges of the image as a result of barrel distortion. These results strictly apply only for the optical configuration employed in our 13-14 August 1996 observing run. They do not necessarily apply for configurations that include the UV-visible dichroic or the tip-tilt secondary mirror.

1 Pixel Scale and Orientation Angle

We observed the Neptune and Uranian systems on 13 and 14 August 1996 with the NSFCAM lens turret set to the nominal 0.148″/pixel mode to insure that both Neptune and Triton could be fit within the same 256x256 pixel image. In this camera configuration, pixel scale and image orientation relative to celestial north were determined from measurements of Uranian satellites positions (Ariel and Umbriel). The absolute image scale was determined from observations on the first night (13 August) and verified by observations on the second night (14 August). Satellite positions were obtained from NAIF ephemeris file URA031, the most accurate available for the 22-Oct-85 through 5-Jan-2000 time period (Jacobson 1991). The 1-σ uncertainties in position relative to the planetary system barycenter for the Uranian satellites is 200 km or about

$0.015''$ (Jacobson, personal communication; Jacobson 1991). On 13 August 1996, the Umbriel-Ariel separation was about $22''$, so that ephemeris uncertainty contributes about $0.07\% \times \sqrt{2}$ to the scale uncertainty. It is not clear to what degree this uncertainty can be reduced by averaging, and over what time interval the errors actually vary. However, given that the pixel scale is consistent within better than 0.1%, it appears that for a series of images taken within an hour or so, the ephemeris error must be considered a bias error, and applied after random errors are averaged.

A sample image of Uranus is provided in Fig. 1. Three satellites can be seen within the camera frame, Ariel (the brightest), Umbriel, and Miranda (the dimmest). The precision with which the satellites can be located is a function of the image brightness, as determined from Monte Carlo simulations of the fitting process. For the specific conditions of this image we estimate uncertainties of 0.014 pixels for Ariel, 0.019 pixels for Umbriel, and 0.088 pixels for Miranda. By far the most accurate image scale is obtained from the Ariel-Umbriel difference because the high brightness of these satellites yields the best positional accuracy. All scale determinations were thus obtained from Ariel and Umbriel, using 15 images in 5 different filters. In each of these images we first measured the coordinates of the satellites using two dimensional gaussian fits, then corrected the coordinates for distortion, as described in Section 2, and determined the distance between them in corrected image pixels. We then determined the Ariel-Umbriel position angle relative to the image y axis, from which we computed the counterclockwise angle from Celestial North to the camera Y axis. The results are shown in Fig. 2, where we find a somewhat surprising dependence of image scale on imaging filter. Note that the average J-filter (+ blocker) scale of 0.1470 ± 0.0001 is nearly 0.6% smaller than the average K-filter scale of 0.1479 ± 0.0001, which is a 9-σ difference. While there is somewhat of a trend towards increasing scale with increasing wavelength, that trend is interrupted by the Spencer 1.73-μm filter, for which we find a lower value than at surrounding wavelengths. ⇐*Fig. 1* ⇐*Fig. 2*

The average scales derived from the satellite measurements on 13 August 1996 are shown in Fig. 3, in comparison with those derived using paired star observations. The latter results were obtained by measuring interstellar angular separations at a variety of wavelengths, then normalizing to the corresponding stellar distances measured in K-filtered images. Using a scale factor determined by χ^2 minimization, we fit this relative function to the absolute scale values derived from the Uranian satellites. The result is shown in Fig. 3. The relative results confirm the smaller scale values for J, H, and Spencer 1.7 filters, and provide additional information for ⇐*Fig. 3*

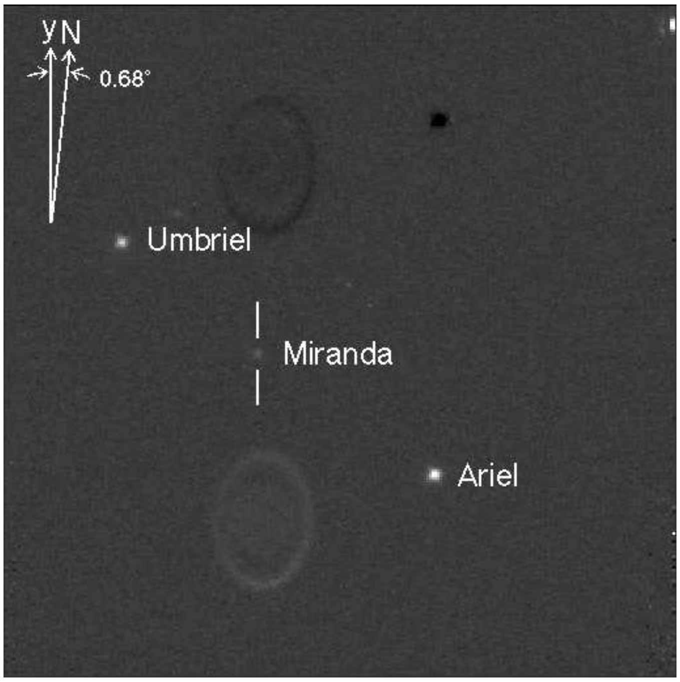

Figure 1: A K-filter Uranian satellite image used to measure the NSFCAM image scale and orientation, taken on 13 August 1996 at 9:07:34 UT with an exposure of 0.5 seconds and 80 coadds. In this image the south pole of Uranus is 87.98° CCW from the image y (up) axis, which is 0.68° CCW from celestial north.

estimating scales for CVF wavelengths of 1.59 and 2.1 μm, for which no satellite observations are available. We used the scaled relative information to adjust the pixel scales as follows. For J, H, Spencer 1.7, and K filters we adopted scales that were weighted averages of the absolute and scaled relative values, with the latter's weighting reduced because of the increased variance associated with the scale factor uncertainty of 0.0001 arcsecond/pixel. For all the CVF filters we could discern no clear scale dependence on filter, and thus adopted a single scale value equal to that of the K filter, which is consistent also with the relatively accurate determination for the 2.00-μm filter. We discovered a significant scale difference for the J filter: with the blocker in place, the J filter scale increases by 0.33%. The observations of the Uranian satellites on 13 August and the star observations on 14 August both used a blocker, while the 14 August 96 Uranian satellite observations did not use a blocker with that filter. The adopted values are summarized in Table I and shown as circles in Fig. 3. The variability of these results from one image to the next is smaller than might be expected from the 1-σ ephemeris uncertainty. In ⇐*Table I.*

fact, measurements of image scales on 14 August 96, not used in formulating the adopted values (except for the J w/o blocker value), serve to confirm those values with very high accuracy (see Table I). Note that the K and Spencer 1.7 values are indistinguishable from the adopted scales.

Table I: Measured and Adopted NSFCAM Image Scales in arseconds/pixel.

Filter	From Star Pairs[1,2]	From 13 Aug 96 Uranian Satellites[3]	From 14 Aug 96 Uranian Satellites[3]	Adopted Values.
J + Blocker	0.14736±0.00020	0.14698±0.00010		0.14699±0.00010
J w/o Blocker			0.14747±0.00005	0.14747±0.00010
H	0.14792±0.00010	0.14743±0.00010	0.14731±0.00002	0.14746±0.00010
CVF 1.59	0.14753±0.00024			0.14791±0.00010
Sp. 1.7	0.14857±0.00019	0.14711±0.00010	0.14708±0.00004	0.14711±0.00010
CVF 2.00	0.14786±0.00057	0.14790±0.00020		0.14790±0.00010
K	0.14780±0.00022	0.14792±0.00010	0.14791±0.00004	0.14791±0.00010
CVF 2.07	0.14770±0.00035			0.14791±0.00010
CVF 2.10	0.14805±0.00030			0.14791±0.00010

NOTES: 1. For scale factor of 0.1478±0.0001″/pixel.
2. Uncertainties listed here are in addition to the above scale factor uncertainty.
3. Includes distortion corrections.

These results are consistent with the Baines *et al.* (1998) value of 0.1478±0.0010 arcsecond/pixel, though our values are at least ten times more precise, and thus are able to clearly resolve the filter dependencies.

From Ariel and Umbriel we also determined camera orientation, specified as the counterclockwise angle from celestial north to the image y axis. As is apparent from Fig. 2, there is relatively little variation in orientation angle that can be associated with filter changes. For 13 August 1996 we found an average orientation angle of 0.686±0.008°, and for 14 August 1996 we obtained 0.664±0.005° using only the more accurate values. The difference of 0.024±0.009° is larger than suggested by our estimated uncertainties, but is still well within our required accuracy of 0.04°. We also used the beam switch direction to estimate the image orientation, and obtained a similar value. For the most part, deviations among various scale and angle determinations, as evident in Fig. 2, seem to be consistent with our estimated uncertainties.

It is important to note that all of the scale determinations account for NSFCAM image distortions that we determined from observations of star fields, as described in the following section. Without such corrections apparent scale variations of several tenths of a percent are possible.

We have ignored possible changes in scale associated with focus variations that occur during an observing run. This is justified by observations indicating no significant dependence of scale on focus position. During the 14 August 96 IRTF observing run we acquired a set of images containing 5 stars in addition to Neptune and Triton. This series included J-filtered images taken during focus adjustment. We measured distances between stars in each image, and analyzed how those distances varied with focus position. We found that the variations with respect to focus were essentially random variations of about the magnitude expected from our uncertainty estimates, and generally was well within 0.1% of the mean. There was no discernible trend indicating a dependence on focus, at least within the range that might be experienced in a night of observing.

The effects of atmospheric refraction on apparent zenith, as given for example by Allen (1964), scaled by a factor of 0.6 to account for the reduced atmospheric mass above 14,000 ft, yields a deviation of 60 arc seconds at 2 air masses. The differential refraction, which has a direct effect on the apparent image scale in the altitude direction, is 0.07% at visible wavelengths and slightly smaller at near IR wavelengths, and in any case not very significant. As most quality observations are nearer to 1.4 air masses, where the effect is at 0.035%, and completely ignorable.

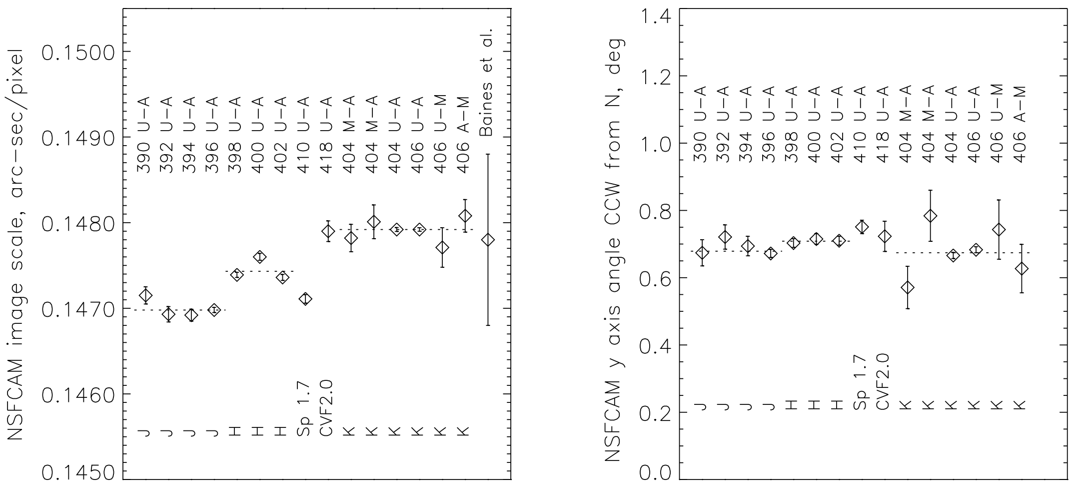

Figure 2: NSFCAM image scale and orientation derived from Uranus satellite observations. The upper labels indicate image file number and satellite combination (U, A, and M denote Umbriel, Ariel, and Miranda) The lower labels identify filters. For these data the J filter was used with a blocker (see next figure for scale results when a blocker is not used. The Baines *et al.* (1998) value was derived from 1995 images.

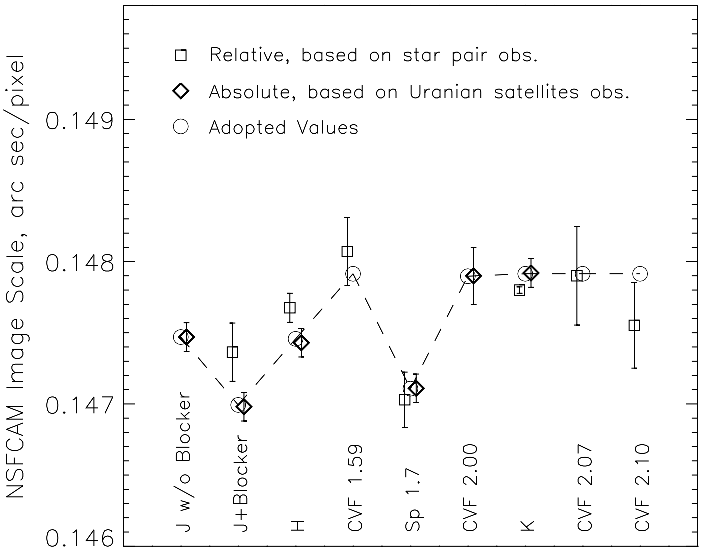

Figure 3: IRTF NSFCAM image scale vs. filter, from Uranian satellite observations (diamonds), from scaled star pair relative observations (squares), and the adopted values from Table I (open circles connected by dashed line). The various symbols are horizontally offset for clarity.

2 Correction of NSFCAM image distortions

Because our method of navigating IRTF images of Neptune requires precise offsets from Triton, we found it necessary to determine and correct for NSFCAM image distortions. We were able to address this issue using the 14 August 96 data set containing multiple star images. In this case we analysed how interstellar distances differed in two different beam positions separated by about $10''$. Because a pointing change of $10''$ moves the stars to substantially different locations on the array, image distortions are revealed by changes in interstellar distances with pointing direction. Our data set is sufficient to constrain only a relatively simple correction function. We assumed that corrected coordinates (x', y') could be written as

$$(x', y') = (x_o, y_o) + (x - x_o, y - y_o)(1 + k(r/r_r)^2) \tag{1}$$

where (x, y) are the raw coordinates, (x_o, y_o) is the center of distortion (near the center of the image), $r^2 = (x - x_o)^2 + (y - y_o)^2$, and r_r=127 pixels. This provides a radially directed correction that increases quadratically with distance from the center, with proportionality constant k. This is consistent with the classical form of distortion derived in third-order aberration theory (Born and Wolf 1980), for which the aberration itself (difference between ideal and actual image coordinates) is proportional to the cube of the off-axis distance. To determine the coefficient k and the effective central coordinates (x_o, y_o), we selected those values which minimize $\chi^2 = \chi_x^2 + \chi_y^2$, where

$$\chi_x^2 = \sum_{i=1}^{N} (dx_a(i,k) - dx_b(i,k))^2 / (\sigma_{x,a}^2 + \sigma_{x,b}^2) \tag{2}$$

$$\chi_y^2 = \sum_{i=1}^{N} (dy_a(i,k) - dy_b(i,k))^2 / (\sigma_{y,a}^2 + \sigma_{y,b}^2) \tag{3}$$

in which $dx_a(i,k)$ and $dx_b(i,k)$ are x-components of the vector connecting the ith star pair in beam positions a and b respectively, and $\sigma_{x,a}$ and $\sigma_{x,b}$ are the uncertainties in those components. Analogous definitions apply to the y-equation. Whether the solution obtained yields a significant and useful correction can be assessed by the value of χ^2 at the minimum. A value close to N indicates that we have removed all the variance beyond what was expected just from the uncertainty in fitting gaussians to the star images. A larger value indicates that the correction function is not doing a good job of characterizing the variations that are present.

The results of our fit to 17 quartets of measurements on 17 star pairs in two different beam positions are shown in Fig. 4. We found best fit parameter values of $k = 0.0055 \pm^{0.0006}_{0.0008}$, $x_o = 137.5 \pm^{3.3}_{3.9}$, and $y_o = 138.2 \pm^{4.3}_{2.1}$. The corrected coordinates yield a factor of six reduction in χ^2 and nearly factor of three reduction in the standard deviation of differences from the desired zero value. The χ^2 minimum value is about 3-5 times N, indicating that the total noise in the measurements is a factor 1.7-2.2 greater than indicated by our uncertainty estimates. Because our estimates are well founded and verified by observational checks, this seems to imply that the distortion corrections themselves have a variability that is not fully characterized by the functional form we have chosen to fit. The residual variation, though rather small, is of the order of 0.1 pixels, and represents one of the larger contributors to the error budget in our offset navigation method. The sign of k being positive means that the NSFCAM image has a barrel distortion, so that features at the edge of the frame appear too close to the center of the frame. $\Leftarrow$*Fig. 4*

In our applicationi this correction is used as follows. We first determine Triton's coordinates within an image containing Neptune and Triton. Next we correct the Triton coordinates to values that would apply to an undistorted image. Using the image orientation and ephemeris values for the angular offsets between Triton and Neptune at the central time of the image, we divide by the central image scale to determine the corrected pixel offset, and add that to the Triton coordinates to obtain the corrected position of Neptune's center in an undistorted image. These coordinates are then distorted by the inverse of the correction function, to define the position of Neptune's center in the distorted image.

The local image scale in the distorted NSFCAM image varies with position. It is smallest near the center, and increases towards the edge, i.e. there are more arcseconds per pixel at the edges of the image. Unfortunately the local scale is not isotropic. The radial scale is given by $S_o(1+3k(r/r_c)^2)$ and the tangential scale by $S_o(1+k(r/r_c)^2)$, where S_o is the filter-dependent adopted central scale at (x_o, y_o), in arcseconds/pixel as given in Table I. To preserve proper angular area, we use the geometric mean scale which is approximated by

$$S(r) = S_o(1+2k(r/r_c)^2) \tag{4}$$

At $r = 127$ pixels, this evaluates to $S(r)/S_o = 1.01$, while at a more typical distance of 60 pixels, the effect is less than 0.25%. In any case, the local scale is taken into account in transforming image coordinates to planetary coordinates.

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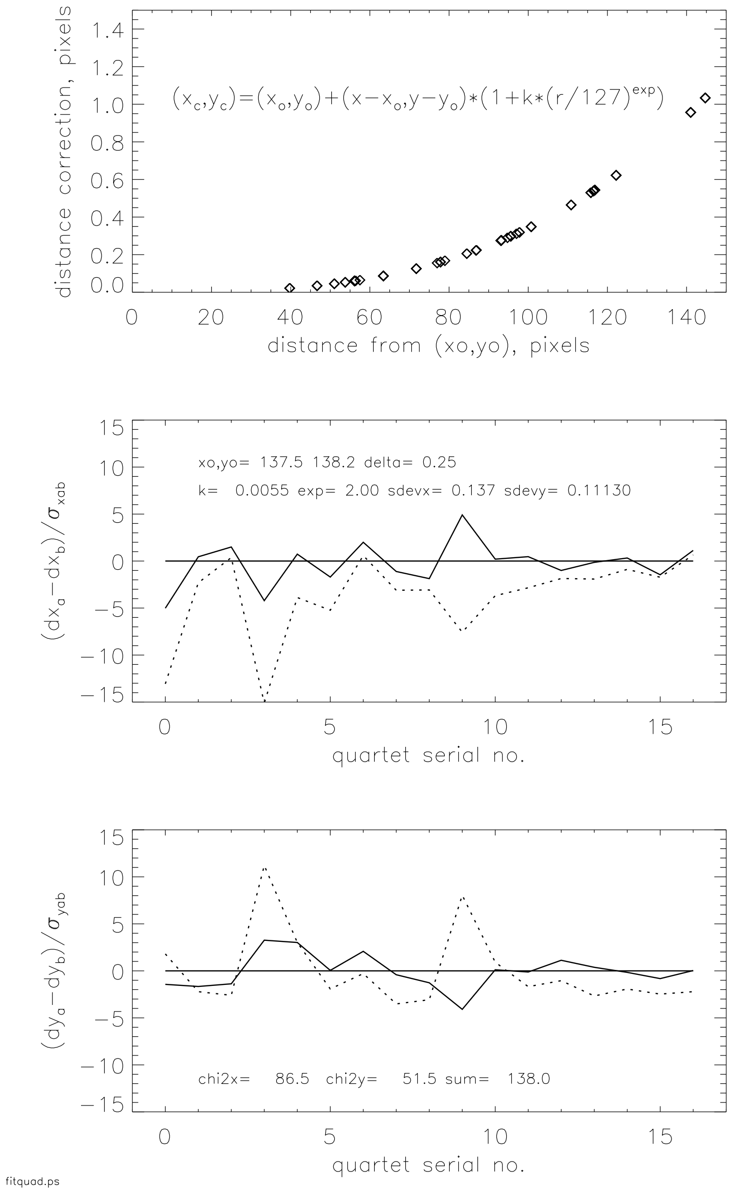

Figure 4: Distortion correction of IRTF NSFCAM images, as derived for use with the 0.15″ lens in 1996 observations. Star pair a-b differential x and y distance measurements divided by expected difference accuracies are shown in lower two panels, where the dashed line shows these ratios when no distortion correction is applied, and the solid curve shows what is obtained with the distortion correction that minimizes $\chi_x^2 + \chi_y^2$. The top panel shows the magnitude of the derived distortion correction as a function of distance from the center of distortion, given by (x_o,y_o).

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